Using simulation to quantify uncertainty in ultimate-pit limits and inform infrastructure placement

by M. Deutsch, E. González and M. Williams

Abstract ■ Uncertainty exists in ultimate-pit limits due to geologic variation and unpredictable economic landscapes. In this work, we show how this uncertainty affects the ultimate pit and how it can be analyzed to improve the mine planning process. A stochastic framework using geostatistical simulation and parametric analysis was used to model the effects of geologic and economic variations on ultimate-pit limits and overall project economics. This analysis was made possible by a new pit optimization implementation that can be automated and set up to calculate ultimate pits for hundreds of different scenarios in a matter of hours. Quantifying ultimate-pit uncertainty early in the mine planning process allows mining engineers to make informed decisions regarding infrastructure placement and to mitigate the possibility of incurring substantial costs to relocate critical mine facilities.

Mining Engineering, 2015, Vol. 67, No. 12, pp. 49-55.
Official publication of the Society for Mining, Metallurgy & Exploration Inc.

Resumen ■ Existen dudas sobre los límites del pit final por causa de la variación geológica y lo impredecible de los paisajes económicos. En este trabajo les mostramos como esta incertidumbre afecta el pit final y como puede ser analizada para mejorar el proceso de planificación de la mina. Se usó un marco estocástico con simulación geoestadística de análisis paramétrico con el fin de poder hace la modelación de los efectos de las variaciones geológicas y económicas en los límites de pit finales y proyectos económicos en general. Este análisis fue posible gracias a una nueva implementación de optimización de pits que puede ser automatizada y configurada para calcular pits finales para cientos de escenarios diferentes en cuestión de horas. La cuantificación temprana de la falta de certeza en pits finales durante el proceso de planificación permite a los ingenieros de minas tomar decisiones informadas en lo que respecta a la ubicación de infraestructura y en la mitigación de posibles costos sustanciales por causa de la reubicación de instalaciones clave de la mina.

Introduction

The mining industry is increasingly concerned about the effects of risk and uncertainty. Uncertain prices, unpredictable global markets, and unforeseeable changes in foreign exchange rates can alter the economic viability of a mining project in substantial ways. This economic uncertainty is compounded by geologic uncertainty. The extent and quality of any given deposit cannot be fully measured and is not known before consequential decisions must be made. These two sources of uncertainty are responsible for the majority of deviation between what happens during operation and what was initially planned. Understanding and explicitly quantifying this uncertainty will lead to better decision making and allow mining engineers and investors to be aware of what may occur.

It is unrealistic to think that one model, estimated from sparse measurements, would be enough to capture the uncertainty from both geologic and economic sources and allow for optimal decisions. The entire breadth of uncertainty should be considered, as both upside and downside risks can have large impacts on the investment potential and operational efficiency of mining projects. The workflow presented here uses Monte Carlo simulation and parametric analysis together to explicitly analyze the breadth of uncertainty from geologic and economic sources as it applies to ultimate-pit calculation and long-range mine planning.

In openpit mining, the ultimate pit represents the limit of extraction

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Paper number TP-14-057. Original manuscript submitted November 2014. Revised manuscript accepted for publication June 2015. Discussion of this peer-reviewed and approved paper is invited and must be submitted to SME Publications by Mar. 31, 2016.

such that mining any more material would require the removal of so much waste as to make any extra ore irrelevant. The ultimate pit is used to assess the economic viability of the project and to guide the mine planning process. It is generally the first stage in overall site planning as other infrastructure will be placed to avoid intersecting the pit limits and sterilizing the ore. The ultimate pit is based on geotechnical, geologic and economic parameters. All of these parameters are uncertain due to sparse measurements, uncertain markets and other risks. These parameters have complex and nonlinear effects on the ultimate pit, which motivate the use of Monte Carlo simulation.

In this paper, we propose a workflow for explicitly analyzing geologic, geotechnical and economic uncertainties as they affect the ultimate pit in order to better understand the risks associated with any given mining project. The workflow allows for the creation of various figures and maps that summarize the risks and allow for risk-qualified decision making. Ultimate-pit uncertainty is also translated into a probability model that is useful in both mine design and project evaluation. This workflow is made possible by an ultimate-pit calculator that is able to analyze hundreds of different possibilities in a few hours instead of a few days or weeks.

We conduct a case study using the workflow and compare the results with those of conventional estimation-based techniques. We then conduct a brief comparative study of ultimate-pit optimization algorithms.

Literature review

The ultimate pit represents the final pit contour such that all economic ore is extracted and all unnecessary waste is left in place. Hochbaum and Chen (2000) formally expressed this problem as:

$$\text{Maximize} \quad \sum_{i \in V} b_i \cdot x_i \qquad (1)$$

$$\text{subject to} \quad x_j - x_i \geq 0 \quad \forall (i,j) \in E \qquad (2)$$

$$0 \leq x_i \leq 1 \quad \text{integer}, \ i \in V \qquad (3)$$

where the block model of the deposit has been re-expressed as a directed graph $G = (V, E)$, in which each block is a node in V. Dependencies, dictated by geotechnical constraints, are expressed as edges in the set E. The economic block value b is used to determine the integer vector x, which indicates if a given block is extracted or left in place.

Equations (1)-(3) have traditionally been solved using the Lerchs-Grossmann algorithm, introduced by Lerchs and Grossmann in 1965. In their paper, Lerchs and Grossmann indicated that the ultimate-pit problem could be expressed as a flow problem but recommended their direct approach, possibly due to computer memory constraints. Picard (1976) provided the mathematical justification by proving that the selection problem is equivalent to computing the maximum valued closure of a directed graph. As a consequence, sophisticated network flow algorithms can be used in place of the Lerchs-Grossmann algorithm, and they can calculate identical results in a fraction of the time. The push-relabel algorithm of Goldberg and Tarjan (1988) and the pseudoflow algorithm of Hochbaum (2001, 2008) are two such sophisticated alternatives.

Hochbaum and Chen's study (2000) showed that the push-relabel algorithm outperformed the Lerchs-Grossmann algorithm in nearly all cases. When the number of vertices is large, greater than a million, network flow algorithms perform orders of magnitude faster and compute precisely the same results.

Parametric analysis. Parametric analysis, introduced by Lerchs and Grossmann in the same 1965 paper, is a technique to approximate an optimum mining sequence by calculating several nested pits. This is commonly called the nested Lerchs-Grossmann algorithm. The block values are decreased by some constant, and Eqs. (1)-(3) are solved again. This reduction serves to constrain the volume of the pit and generate a smaller nested pit. When this process is repeated, several nested pits are generated that, when taken as a sequence, extract the highest valued blocks first. However, reducing the block values by a constant does not have an intuitive relationship with the inputs to the block value calculation, so an alternative reduction strategy is generally employed.

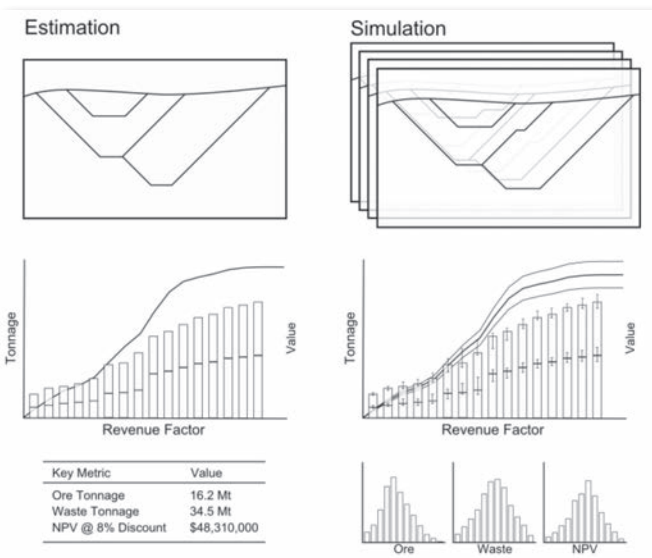
Matheron (1975) introduced a form of parametric analysis where the block value b is expressed in terms of a parameter λ as in:

$$b_i = \lambda \cdot c_i + d_i \qquad (4)$$

where c_i is the sum of the terms linearly dependent on λ and d_i is the sum of the independent terms. In practice, d is taken to be the costs associated with extracting, processing, transporting and selling the block, and c is taken to be any revenue. In this case, λ is called a revenue factor after Whittle (1989). The ultimate pit is calculated

Figure 1

Results of standard estimate-based long-range planning (left), and how simulation changes the results to account for uncertainty (right).



for many revenue factors $\lambda \geq 0$ to generate the nested pits.

Geostatistics and simulation. Determining the values of the *c* and *d* terms in Eq. (4) is both site and commodity specific and depends on many different parameters. Many of the parameters are local, in that they vary by location and depend on some geologic attribute such as metal content, rock type and specific gravity. These geologic attributes must be known at every location within the volume of interest to inform the economic block value, but they cannot be directly measured at every location. Therefore, geologists and mining engineers have turned to the field of geostatistics to inform robust interpolation and extrapolation techniques to fill in the gaps. These techniques are based on the sound application of geology and statistics to generate fully sampled models that can then be used in downstream studies. The theory and modern practice of mining geostatistics are discussed in Rossi and Deutsch (2014).

Estimation-based techniques such as inverse distance or kriging are only capable of providing one model that is smooth by construction and possibly systematically biased. Geologic uncertainty cannot be captured with a parameter and therefore a single geologic model is not enough. Instead, geostatisticians have adopted a stochastic framework based on Monte Carlo simulation. Simulation techniques such as sequential Gaussian simulation (Isaaks, 1990) and sequential indicator simulation (Alabert, 1987) are free of conditional bias and provide many different equiprobable realizations that, when analyzed together, sample the underlying geologic uncertainty.

Uncertainty in mine planning. The results of geostatistical simulation have been used to great effect in mine planning before. Dimitrakopoulus et al. (2002) analyzed the effect of geologic uncertainty on the ultimate pit of a disseminated, low-grade gold deposit and found that the realizations departed substantially from the kriged estimate. Dimitrakopoulus et al. (2007) used orebody uncertainty to determine designs that perform well in the presence of uncertainty. They indicated that designs based on stochastic mine planning had led to substantial increases in net present value as the entire range of uncertainty is analyzed.

Monkhouse and Yeates (2005) advocated moving beyond the naive optimization of a single model of the subsurface and instead urged practitioners to use all sources of uncertainty to make better plans and decisions. Plans that use uncertainty can be developed to achieve optimal results across a reasonable range of real-world inputs.

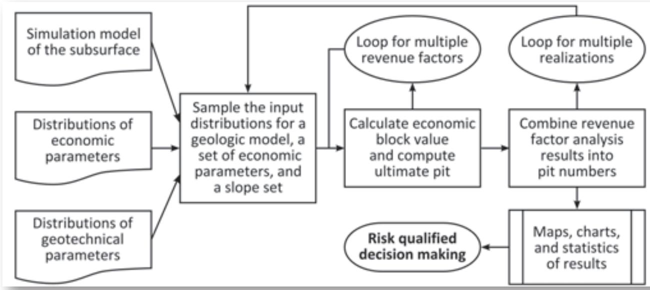
Even with these previous studies there is more to be done to analyze uncertainty in the ultimate pit. One of the drawbacks to sensitivity studies and simulation in general is the extra computation time and professional time required. We have, therefore, developed a workflow that can be used to capture this uncertainty and quickly summarize it using commercially available software.

Workflow

The typical results of a long-range mine planning exercise for a feasibility study, or for analysis during production, include a series of pit shells, a pit-by-pit graph and a table of metrics for the chosen ultimate pit. Traditionally, these results are based off of a single estimated model and therefore have no consideration of geologic uncertainty. To add more information to these results and account for all sources of uncertainty, stochastic methods are used. A conceptual depiction of this is shown in Fig. 1. The pit shells are replaced with many different possible pit shells. Uncertainty in the pit-by-pit graph is depicted by error bars. The various metrics are replaced with histograms showing the distribution. Additionally, a further compilation step is introduced to generate a probabilistic model of the pit shells.

Figure 2

Flowchart showing the proposed workflow for analyzing ultimate-pit uncertainty.



The workflow is shown in Fig. 2. Although the entire workflow can be done manually, this becomes practically infeasible as the number of realizations increases. Simple scripts are used to loop through the realizations and synthesize the results. There are three inputs to the ultimate-pit uncertainty workflow: a geostatistical simulation model of the subsurface; distributions of the input economic parameters; and a distribution of the geotechnical parameters.

To achieve the improvements shown in Fig. 1, a simulation model of the subsurface is required to generate equiprobable realizations of the underlying geology. The geostatistical simulation workflow to generate these results will not be discussed in this paper. However, practitioners of simulation should be mindful of recreating the input statistics, such as distributions, correlations and variograms, and ensuring the validity of the simulation as a whole. Issues of volume variance and data support should be considered, though this may be avoided by simulating at the data scale before averaging to the relevant scale for mine planning.

To analyze uncertainty in the economic parameters, we introduce a stochastic economic block value function. At its core, the economic block value function is simply revenues less costs. A simplistic economic block value formula may have the following form:

$$b_i(\lambda) = \lambda(T_o \cdot g \cdot r \cdot P) - T_o \cdot PC - T \cdot MC \qquad (5)$$

where λ is the revenue factor, T_o is the tonnage of ore, *g* is the grade, *r* is the recovery or percentage of product recovered, P is the commodity price, PC is the processing cost, T is the total tonnage and MC is the mining cost. This is a very basic economic block value function and often more complicated functions are used in practice. Normally, several different possible processes are defined. Selling costs and different mining costs are included. Multiple factors are applied based on rock type, location or other parameters. Geometallurgical attributes, contaminants and other local inputs are also often included.

By using geostatistical simulation, we have accounted for uncertainty in the location-dependent inputs to the economic block value calculation, such as grade, as they vary between geologic realizations. However, some of the global inputs such as commodity price are also variable, and their values are uncertain. To account for uncertainty in these global parameters, we will define some distribution that captures the parameter in question and sample it once for each realization. This leads to a stochastic economic block value function that will have different global economic parameters by realization and therefore account for uncertainty. If production data, or some other information, are available, this may be done explicitly without resorting to assumptions regarding the character of the underlying distribution.

The geotechnical information describes the allowable pit slopes for every block and can take many forms. Commonly, slope requirements are defined by azimuth within different zones. Geotechnical uncertainty can be included by having different slope definitions for each realization or by basing the slopes on zones that had been simulated using some form of categorical simulation. Depending on the geotechnical context of the area in question, it may make sense to hold the slope definitions constant across all realizations.

The remainder of the workflow is to draw from the input distributions, fully sampling the space of uncertainty, and then perform parametric analysis with that particular geologic model, economic block value function and slope definition. The sampling and parametric analysis is then completed for many realizations. After a reasonable number of realizations, on the order of a few hundred, have been completed, the results are synthesized.

To aid in describing how to synthesize the results, we will use the following notation for the pits. Recall that each pit vector, calculated from Eqs. (1)-(3), is an integer array with a 1 for each block that lies within the ultimate pit limits and 0 for any block outside. Denote a single pit vector $x_{l,\lambda}$, where l is the realization index and λ is the revenue factor. Let L and Λ represent the set of realizations and revenue factors, respectively. One common summary is the pit number, which is calculated as follows:

$$PN_i(l) = |\Lambda| - \sum_{\lambda \in \Lambda} x_{l,\lambda,i} + 1 \quad i \in V \tag{6}$$

The pit number is set to 0 for air blocks and to a large number for blocks that are outside the largest pit.

The pit numbers correspond directly to the pit-by-pit graph. Ore and waste tonnages may be calculated simply, and with many realizations, error bars may be added. The error bars indicate the variability in both ore and waste for that particular revenue factor. Histograms of key indicators for any given revenue factor may be extracted and reported. The discounted cash curve and net present value depend on determining an extraction sequence that honors production and extraction constraints. Determining an extraction sequence is beyond the scope of this paper.

A further useful summary of ultimate-pit uncertainty is the probability model. The probability model is similar to the hybrid pits of Whittle and Bozorgebrahimi (2004). It is defined as:

$$PM_i(\lambda) = \frac{1}{|L|} \times \sum_{l \in L} x_{l,\lambda,i} \quad i \in V \tag{7}$$

The probability model indicates what the probability is for a given block to be extracted for a given revenue factor. For example, if it is assumed that the ultimate pit occurs at some revenue factor λ_U, $PM(\lambda_U)$ can form the basis for designing the ultimate pit and the probability models for $\lambda < \lambda_U$ can be used to assist in sequencing the mining process to extract high-probability ore first. The intersection of $PM(\lambda_U)$ and the topography can also be plotted on a map that indicates the range of possible locations of the final pit crest.

Case study

A case study of a small copper deposit was carried out to test the workflow and analyze ultimate-pit uncertainty with real data. The deposit was modeled using both estimation and simulation techniques.

Parametric analysis was completed using stochastic economic block value functions and varying slopes. The results were then synthesized, and ultimate-pit uncertainty was assessed to inform mine valuation and mine planning.

Geologic modeling. The area of interest had 43 drillholes with combined length of 1,450 m, about 400 assays measuring copper content and rock type, and five rock types associated with the host rock, sedimentary layers, quartz, andesite and the high-grade copper-bearing dyke. This is an exploration dataset and the deposit was sparsely sampled, so there was substantial geologic variation. This dataset is simplistic, with only one product, but the workflow is suitably general for more realistic cases.

An implicit model of the rock types using signed distance functions was generated to inform the extent and character of the domains. This model was conditioned to drillhole data and existing geologic interpretation. Within each domain, experimental variograms of copper grade were calculated and modeled. These variogram models were then used to generate a best-estimate model using a standard ordinary kriging workflow. Cross validation was completed, and the parameters were tuned to form a conventional "best" estimate.

The copper assays were then transformed to facilitate sequential Gaussian simulation. Normal score variograms were calculated and modeled, and copper grade within the domains was simulated. Several of the realizations were checked visually and many more were assessed for variogram and histogram reproduction. Two of the simulated realizations are shown in Fig. 3. These realizations form the basis for the geologic uncertainty.

Ultimate-pit calculation. For the case study, the basic economic block value function defined in Eq. (5) was used. Lacking production data and any other insight, the various global parameters were assumed to be normally distributed with the means and standard deviations given in Table 1. These assumptions were made without loss of generality; if more detailed information existed, it could easily be implemented into the scripted workflow. In practice, a more realistic economic model should be used that varies with time and considers how price uncertainty increases into the future.

Forty-six revenue factors uniformly distributed between 0.3 and 1.2 were used in the parametric analysis. These nested pits formed the basis for the pit-by-pit graph and the sequence used to generate the discounted cash flows. A high-level sequence was calculated using an assumed mining rate of 500 kt/a (551,000 stpy) and a bench lag of three benches.

A single realization consists of one geologic model generated using sequential Gaussian simulation, and a set of parameters sampled from Table 1. Five hundred such realizations were calculated. The entire process required the calculation of 23,000 ultimate pits on a model with just over 1.8 million blocks.

Results and discussion

Figure 4 shows the pit-by-pit graph and distributions of several key performance indicators. There is substantial variation in the key performance indicators across all revenue factors. The ore and waste tonnage bars indicate the mean value across all of the realizations, and the error bars indicate the 10th and 90th percentiles. The discounted cumulative cash flow is shown as three lines with the bold line indicating the mean and the two surrounding lines as the 10th and 90th percentiles. The dashed black line shows the conventional estimation-based results using the kriged model and performance indicators calculated with this single model.

The variation in the key performance indicators emphasizes the need to consider uncertainty in parametric analysis and ultimate-pit calculation. Strategic decisions are based on the value and extent of the ultimate pit and if those values are demonstrably variable, those decisions should adapt. It is one thing for a mining engineer to decide on the fleet to purchase based on a single number, but if the distribution is known, the fleet can be purchased with the appropriate amount of flexibility in mind.

In this case study, the average kriging model with average economic parameters does not give an average assessment after the ultimate-pit and long-range planning analysis is completed. The expected discounted value across all realizations is $11.3 million with a standard deviation of $2 million, and the kriged model indicates a value of $9.7 million. Deviation is expected, as the ultimate-pit calculation is not linear with respect to the input parameters and average inputs do not guarantee average outputs. However, it is not guaranteed that the average assessment will be lower than the average. This discrepancy is, in part, due to the nonlinear nature of ultimate-pit optimization and the smoothing effects of kriging. In this case study, the cutoff grade is close to the mean, which

Figure 3

Two realizations of copper grade. Blocks are displayed semi-transparent and colored by copper content.

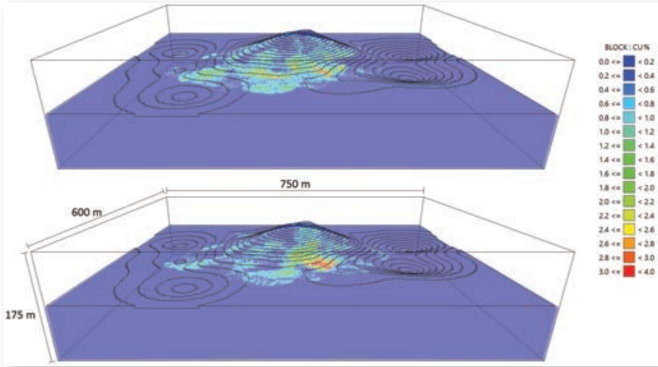


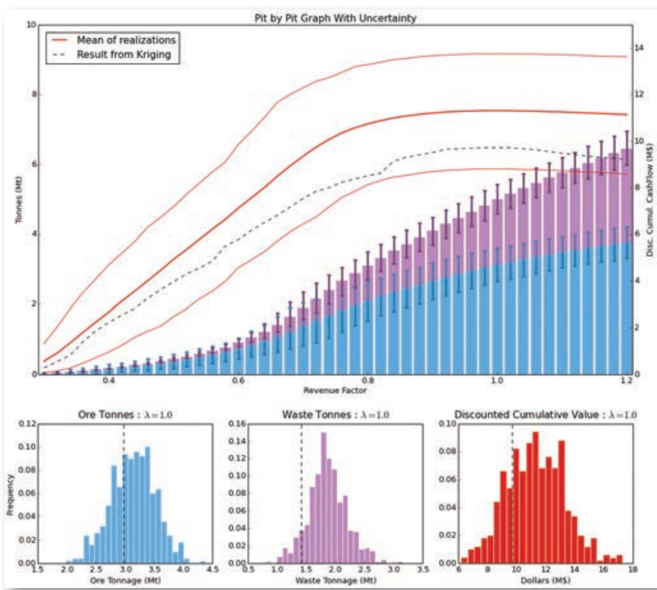
Table 1

Global parameters varied in the case study.

Parameter	Mean	Standard deviation
Mining cost	$2/t	$0.2/t
Recovery	75%	1%
Price	$2.2/lb	$0.2/lb
Processing cost	$4.8/t	$0.1/t
Overall pit slope	45°	1°

Figure 4

Results of the case study for ultimate-pit uncertainty. Error bars and lines indicate the 10th and 90th percentiles. The histograms are for revenue factor of 1.0. The dashed line shows the result from a conventional estimation-based workflow.



causes slight variations in grade to substantially alter the economic block value and therefore change the pit.

A probability model was extracted for revenue factor 1, and the intersection of this model with the topography is shown in Fig. 5. Every block is colored based on its likelihood to be within the pit. The red innermost blocks are in all 500 pits, the blue outside blocks are in none of the pits. The pit crest of the single model from kriging is also shown as a thick black line. This model indicates where the crest could be, based on the underlying uncertainty. From a mine-planning perspective, a continuum of results is much more valuable. In this case study, the pit wall is much less variable along the east side, but there are a great many pits that extend toward the west. Infrastructure can now be placed appropriately, accounting for where the pit may be in the future.

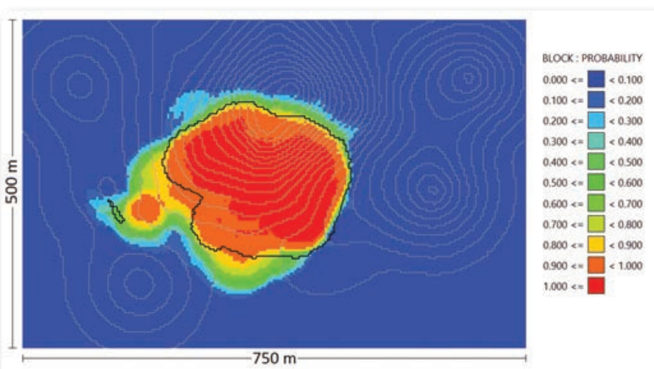
Algorithm comparison

Running sensitivity studies in the presence of geologic, economic and geotechnical uncertainty requires a great deal of computing power, and the choice of the underlying algorithm, as well as the implementation itself, is vitally important to facilitate this workflow. Three programs that can be used to solve the ultimate-pit problem were considered in a runtime comparison: Maptek's Vulcan 9.1 lg_optipit (Maptek, 2015), which uses the push-relabel algorithm; the Centre for Computational Geostatistics' lg3d (Deutsch and Deutsch, 2014), which uses the Lerchs-Grossmann algorithm; and Hochbaum's pseudo_fifo v3.23 (Hochbaum, 2008), which uses the pseudoflow algorithm.

These programs are quite different in terms of scope and utility. Pseudo_fifo is primarily a computational utility implemented as a general solution to the underlying "max flow/min cut" problem and does not build the precedence graph, so lg3d was used to create the graph for pseudo_fifo. The Centre for Computational Geostatistics' lg3d was written with geostatistical sensitivity in mind and runs multiple realizations in parallel and shares the precedence graph between realizations. Maptek's Vulcan 9.1 lg_optipit has more options than lg3d and is a commercial implementation that uses a binary format for storing the block model, enabling faster retrieval.

Figure 5

Intersection of the probability model for revenue factor = 1 with the surface. The pit crest for the estimated model is shown as the thick black line.



Eleven models from five datasets were compiled and run against each program. For each dataset, Table 2 lists the number of blocks, which is a tangible metric and highly familiar but can be misleading depending on the number of air and waste blocks as well as where the ore blocks are. Therefore, the graph size is also reported in terms of $|V|$, the number of active vertices, and $|E|$, the number of edges following basic graph trimming. Active vertices consist of all ore blocks, as well as any waste blocks supported by any ore block, but do not include specific air blocks that can be safely ignored. In all of the test cases, eight benches of edges were used with a pit slope of 45°.

Table 2 also shows the solution times, measured from when the graph was defined and allocated until the block selection was complete, which for the flow-based algorithms was once the minimum cut had been calculated and for the Lerchs- Grossmann was once all strong nodes corresponded to a valid closure of the precedence graph. The flow-based algorithms, pseudoflow and push-relabel, were faster than the Lerchs-Grossmann.

While the numbers of active vertices and edges had a strong impact, there were other underlying complexities that greatly affected the solution time. The Copper Pipe dataset had fewer than half the nodes and edges of the Gold Vein dataset, yet took much longer to solve for all of the algorithms. This is possibly due to the steep vertical configuration of the ore blocks, which led to a much more difficult selection.

This brief run-time comparison indicate that if the problem size is under a few hundred thousand active vertices, it is not overly consequential which engine is used, but as the number of blocks increases, and thereby the numbers of vertices and edges, flow-based algorithms are superior due to their lower algorithmic complexity. After several million blocks, the Lerchs-Grossmann algorithm is not appropriate, and reblocking must be used to reduce the problem size, especially when many realizations must be computed. There are other practical considerations, such as the ease of scripting, if realizations can be completed in parallel, if the expensive graph-building process can be completed once, and the possibility of integration with existing commercial solutions. With the flow-based algorithms, the solution time is no longer the bottleneck. Reading the data and building the graph dominate the wall time, which is precisely where an integrated solution excels. In practice, flow-based algorithms are better than the Lerchs-Grossmann algorithm for computing ultimate pits in the presence of geologic, economic and geotechnical uncertainty due to their greatly reduced runtime.

Conclusions

We proposed a workflow to capture ultimate-pit uncertainty using Monte Carlo simulation and an efficient ultimate-pit solver. The results sample the entire space of uncertainty and allow for risk-qualified decision making. Variability in the subsurface and all other input parameters is explicitly translated through the long-range mine planning transfer function to analyze uncertainty in the ultimate pit. We explored some practical considerations for synthesizing the results and showed how a probability

Table 2

Model summary and solution times for algorithm comparison.

Dataset				Solution time (sec), average of 5 runs		
Name	No. of blocks	No. of vertices	No. of edges	Push-relabel	Pseudo-flow	Lerchs-Grossmann
McLaughlin*	2,140,342	544,000	56,352,700	20	2	44
Gold Vein	16,244,739	3,523,000	620,740,400	9	4	2712
Case Study	1,827,500	123,400	11,254,700	1	0	16
Copper Pipe	2,754,000	1,436,700	255,802,800	52	9	56,585
Bauxite**	46,800	14,500	813,200	0	0	0
Bauxite**	115,200	35,200	2,779,700	0	0	1
Bauxite**	220,000	65,400	6,117,700	1	0	3
Bauxite**	374,400	121,800	12,913,700	1	0	11
Bauxite**	742,500	231,400	28,113,100	4	0	11
Bauxite	1,760,000	523,800	75,557,700	12	2	104
Bauxite**	2,995,200	910,900	139,738,700	22	5	683

*The McLaughlin dataset is from the Minelib library of openpit mining problems (Espinoza et al., 2012). **Reblocked

model can be generated for infrastructure planning or to direct future drilling activities. We also investigated the performance of three algorithms for computing the ultimate pit.

A case study was completed on a small exploration dataset with 43 drillholes. Kriging and simulation were used to build the geologic models. The models were translated into economic block values using an average function for the kriged model, and a stochastic function for each realization that sampled the underlying economic parameters. Parametric analysis was then completed using a range of revenue factors for all realizations. The results were synthesized into figures and graphs that summarized the risks inherent in the mining project.

The results of the case study indicate a need for explicitly analyzing uncertainty. The mean Monte Carlo result shows a larger pit that generates more revenue and requires more ore and waste to be mined than the conventional analysis based on the mean input parameters would suggest. This discrepancy may lead to suboptimal decisions and plans that do not consider the underlying uncertainty. Using an average model does not guarantee average results with complex nonlinear processes, such as pit optimization and long-range mine planning.

Analyzing uncertainty at an early stage allows for plans to be developed that account for what could occur. Uncertainties in geologic, economic and geotechnical parameters can be quantified and analyzed, which allows for flexible plans to be developed and appropriate risk-qualified decisions to be made. ■

Disclosure statement

Maptek sells Vulcan 9.1 as a commercial software package.

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